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Short Communication

Power flow paths in stiffened plates

X.D. Xu, H.P. Lee*, C. Lu

Institute of High Performance Computing, 1 Science Park Road, ♯01-01 The Capricorn, Singapore Science Park II,
Singapore 117528, Singapore

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Abstract

The structural intensity of a rectangular plate with stiffeners attached is calculated using the finite element method. The power flow behaviors for the plate with stiffeners attached at various positions are investigated. The effects of geometrical properties of stiffeners on the changes of power flow are quantitatively analyzed by numerical integration for the structural intensities. A series of calculation results showed that the total amount of injected or transmitted power flow is dependent on the natural vibration frequency of the whole structure and the relative percentage of power flowing through the cross section of the plate or the stiffener depends on the ratio of their relative flexural rigidity.

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1. Introduction

Stiffened plate construction has been widely used in many engineering structures for its high strength-to-weight ratio. The plate components with stiffeners attached can provide effective strength and resistance to vibrational motions. Understanding the energy flow from a vibration source to other parts of a structure helps an engineer to pinpoint and control vibration problems. Since the stiffeners with different reinforced directions and cross-sectional geometry properties will affect and change the power flow paths in the structure by different manners, the investigation of vibrational power flow paths in stiffened plates together with the effects of stiffeners will be helpful to obtain more reasonable and optimal design resolution.

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*Corresponding author. Tel.: +65-6419-1288; fax: +65-6778-0522.
E-mail address: hplee@ihpc.a-star.edu.sg (H.P. Lee).

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Structural intensity is such a technique that can be used to indicate both magnitude and direction of the power flow through a structure from the excitation source to the dissipation sink. The investigation of structural intensity started in the early 1970s which aimed to develop measurement techniques of structural intensity for simple structural elements [1,2]. The computation of the structural intensity was later developed since 1990 [3–5]. However, most of the calculations have been confined to simple structures such as rectangular plates. Introducing stiffeners to the plate will increase the complicacy on the analysis and calculation of structural intensity for the whole assembly. Hambric [3] considered the structural intensity in a dissipative cantilever plate with stiffeners attached using the finite element method. Rook and Singh [6] developed a computational strategy on the structural intensity calculation for plate-beam structures connected by bearings. Seo et al. [7] presented a power flow analysis method to predict the vibrational response of the reinforced beam-plate structures in medium-high frequencies domains.

2. Results and discussion

The formulae used in the calculations of the structural intensity in the x and y directions are given as Eq. (6) in Ref. [4]. A rectangular steel plate having the length of 2 m, width of 1.5 m and thickness of 0.005 m attached by an unsymmetrical eccentric flat stiffener of cross-sectional dimension of 0.1 m height and 0.005 m thickness is used as the first numerical example. The plate is simply supported along two short edges and free on two long edges. The excitation force is applied near the bottom left-hand corner of the plate with a magnitude of 10 N, and two excitation frequencies are chosen which are close to the first and third structural natural frequencies, respectively. The natural frequencies for two models are illustrated in Table 1. A damper with damping coefficient of 100 Ns/m is attached at the right-up corner of the plate.

Figs. 1(a) and (b) indicate the structural intensity vectors in the plate without and with a stiffener, respectively, under the excitation frequency close to the structural fundamental frequency, where the reference vector scale denotes the magnitude of structural intensity corresponding to 1 cm length arrow. The distribution of energy flow is relatively dispersed in the plate without stiffeners, i.e., the affected area is relatively large. However, the magnitude of energy flow in plate is obviously localized at positions of the source and the sink when the stiffener is linked between them, whereas the affected regions of energy flow from the source to the sink are smaller compared with that without stiffener attached. The stiffener apparently changes the orientations of energy flow in its vicinity. Fig. 1(c) shows the energy flow pattern in the stiffener. Since the same reference scale is selected for showing the structural intensity vectors in the plate

Table 1
Natural frequencies of models (unit: Hz)

Order of N. F.	First	Second	Third	Fourth	Fifth	Sixth
Un-stiffened plate	2.93	6.00	11.82	15.81	16.72	26.74
Stiffened plate	3.22	6.21	12.03	15.90	23.34	29.00

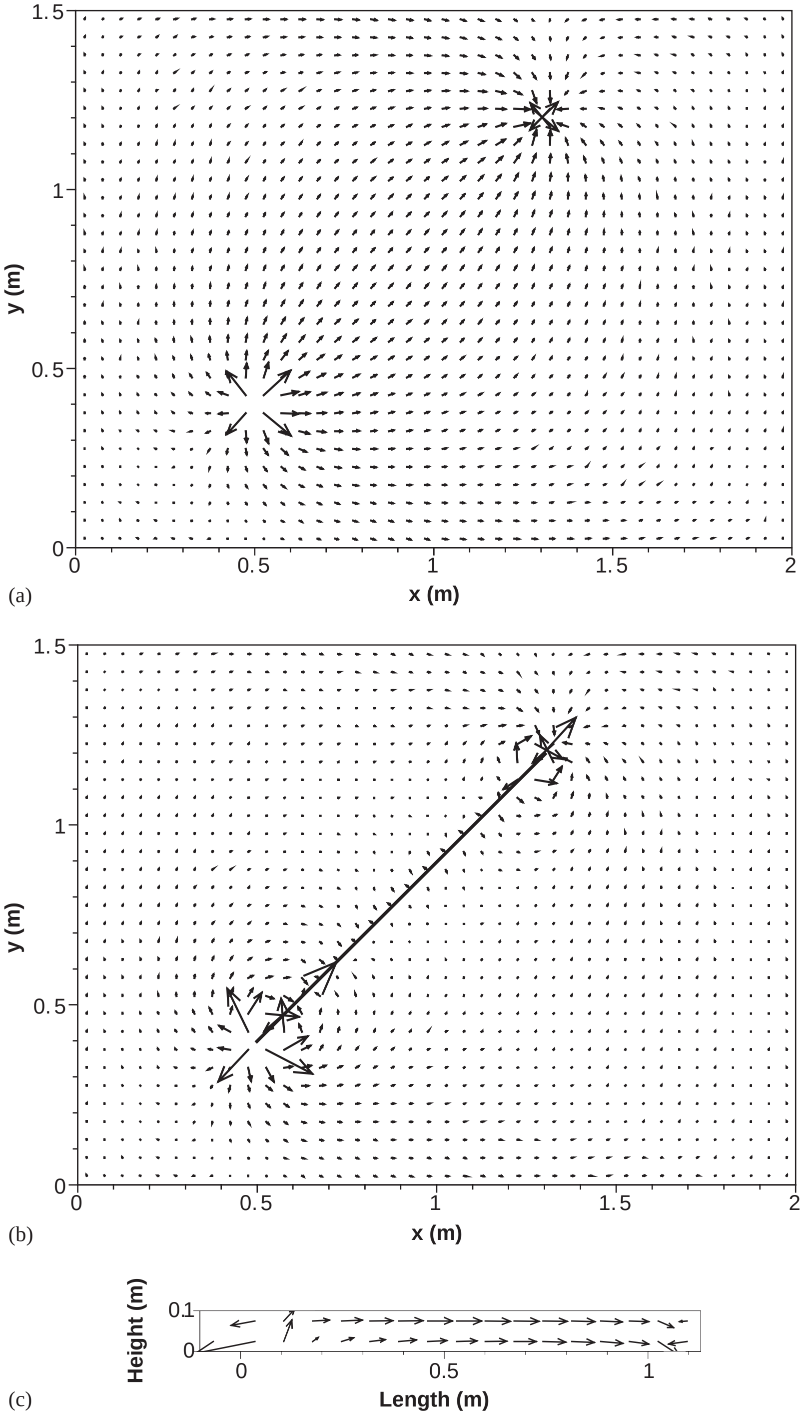


Fig. 1. Structural intensity vectors per unit width at the excitation of the fundamental frequency: (a) vectors in the unstiffend plate (ref. scale = 2 W/m); (b) vectors in the stiffend plate (ref. scale = 0.5 W/m); (c) vectors in the stiffener (ref. scale = 0.5 W/m).

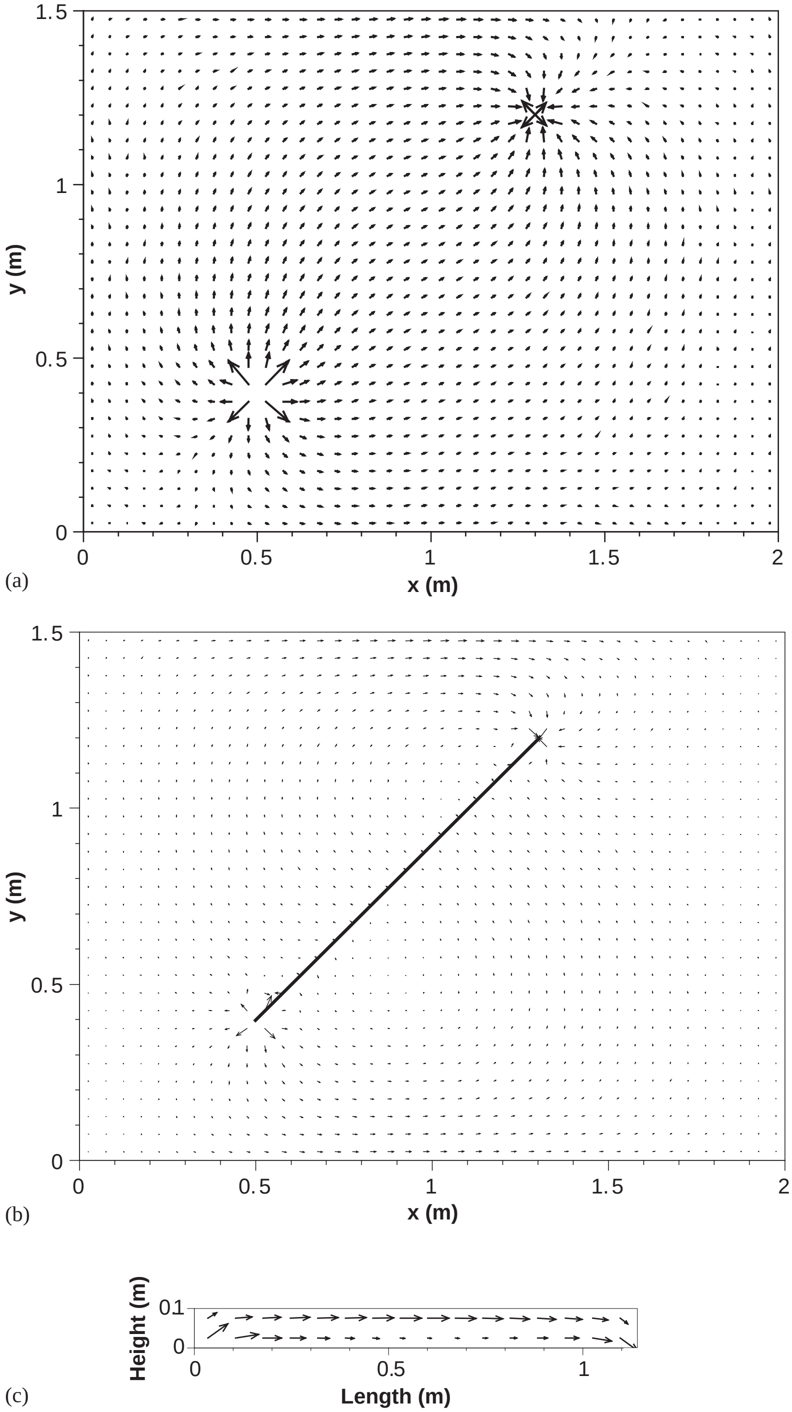


Fig. 2. Structural intensity vectors per unit width at the excitation of the third natural frequency: (a) vectors in the unstiffend plate (ref. scale = 5 W/m); (b) vectors in the stiffend plate (ref. scale = 8.3 W/m); (c) vectors in the stiffener (ref. scale = 8.3 W/m).

and in the stiffener, it can be found that the main energy flow passes from the source to the sink through the stiffener directly. In order to further verify this phenomenon another excitation frequency which is close to the third structural natural frequency is employed. The patterns of energy flow in the un-stiffened plate, stiffened plate and stiffener are depicted in Figs. 2(a)–(c), respectively.

It is more interesting to quantitatively investigate the influences of stiffener's cross sectional geometry properties on the changes of the transmitted power flow. Since the structural intensity in a plate is expressed in the form of power flow per unit width while it is expressed as total power flow over the cross section of a beam, the integration of the structural intensity in the plate must be performed numerically along a certain integration path in order to obtain a consistent energy flow unit.

The calculation model used is similar to the previous one, where a longitudinal stiffener is attached to the plate but it does not directly link the source and the sink in this model. The finite element model is shown as Fig. 3. The positions of the source and the sink and the excitation conditions (magnitude and frequency) remain the same for all cases, the excitation frequency is chosen close to the minimum fundamental structural frequencies of all cases. The solid line represents the stiffener and the dash lines indicate the integration paths which pass along the centers of each element. A series of stiffeners with different cross-sectional properties are selected as the calculation cases. The properties of stiffeners are illustrated in Table 2, where h_s and t_s are the height and thickness of the stiffener, respectively, A_s is the cross-sectional area of the stiffener and A_p denotes the area of the plate per unit width $= t_p/(1-v^2)$, EI_s is the flexural rigidity of the stiffener along the axis of itself, D denotes the plate flexural rigidity per unit width $= Et_p^3/12(1-v^2)$. The first case in the table denotes the un-stiffened plate model.

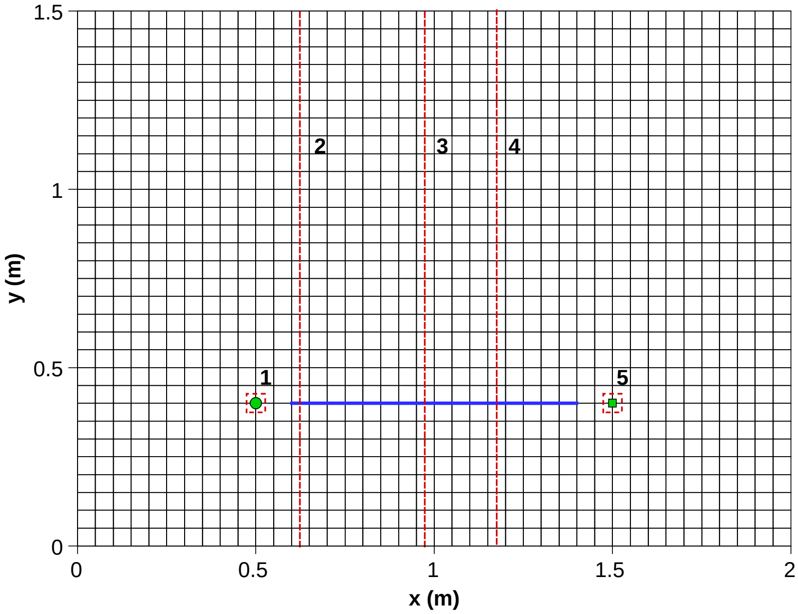


Fig. 3. Finite element model and integration paths.

Table 2
Calculation cases by changing the cross-sectional geometry properties for stiffeners

Case	h_s (m)	t_s (m)	$A_s/A_p *(1-v^2)$	$\mathrm{EI}_s/D *(1-v^2)$	N. F. (Hz)
1	0	0	0	0	2.920
2	0.01	0.005	0.01	0.04	3.002
3	0.02	0.005	0.02	0.32	3.317
4	0.05	0.005	0.05	5	3.711
5	0.1	0.005	0.1	40	3.743
6	0.05	0.01	0.1	10	3.718
7	0.02	0.025	0.1	1.6	3.569
8	0.01	0.05	0.1	0.4	3.294

Table 3
Calculated results of total power flow through different integration paths for different cases (unit: W)

Case	Path 1	Path 2		Path 3		Path 4		Path 5
		Plate	Stiffener	Plate	Stiffener	Plate	Stiffener	
1	0.122	0.112	—	0.112	—	0.112	—	0.122
2	0.470e−1	0.505e−1	−0.681e−2	0.404e−1	0.318e−2	0.405e−1	0.310e−2	0.470e-1
3	0.667e−2	0.867e−2	−0.254e−2	0.404e−2	0.179e−2	0.428e−2	0.157e−2	0.667e-2
4	0.181e−2	0.221e−2	−0.613e−3	0.626e−3	0.768e−3	0.755e−3	0.669e−3	0.181e-2
5	0.164e−2	0.197e−2	−0.528e−3	0.506e−3	0.738e−3	0.628e−3	0.647e−3	0.164e-2
6	0.175e−2	0.214e−2	−0.581e−3	0.570e−3	0.778e−3	0.697e−3	0.681e−3	0.175e-2
7	0.267e−2	0.342e−2	−0.102e−2	0.108e−2	0.107e−2	0.126e−2	0.932e−3	0.267e-2
8	0.715e−2	0.949e−2	−0.288e−2	0.410e−2	0.211e−2	0.439e−2	0.184e−2	0.715e-2

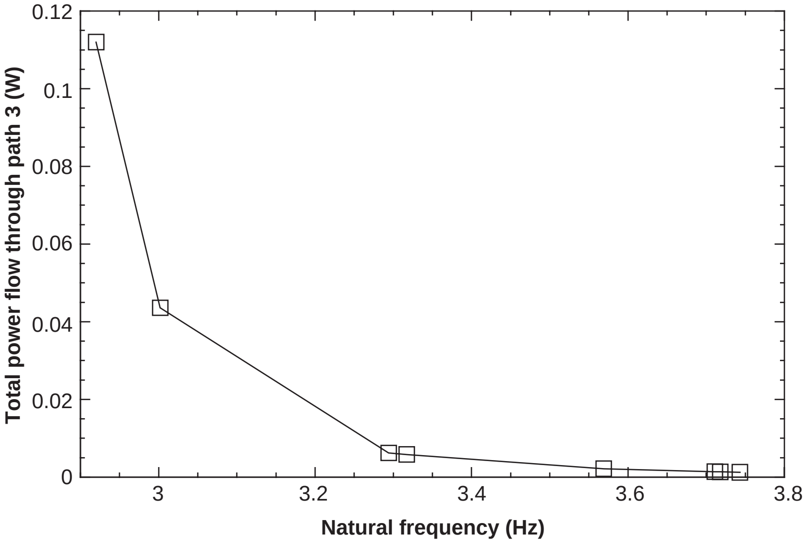
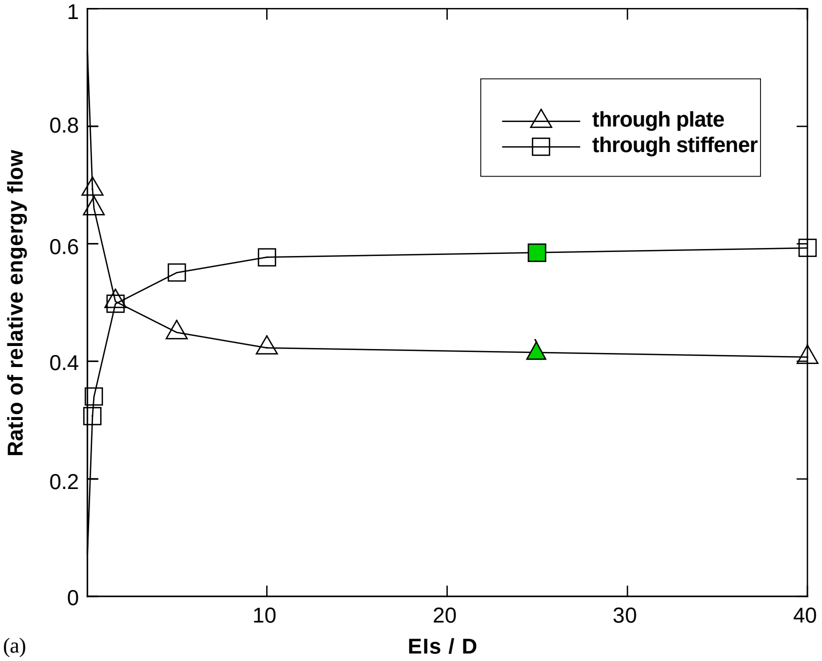


Fig. 4. Total power flow vs structural natural frequency.



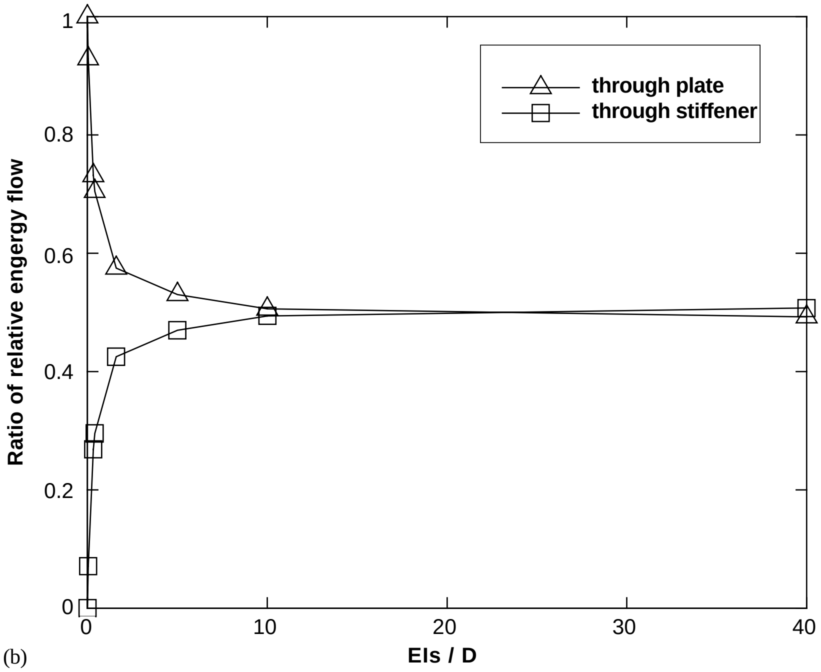


Fig. 5. Ratio of relative energy flow through plate and through stiffener: (a) along integration path 3; (b) along integration path 4.

The calculated results of total power flow transmitting through different integration paths are illustrated in Table 3 for different example cases, in which the value of total structural intensity in the plate plus that in the stiffener indicates the total power flow through the structure. It can be found that the amount of energy injected by the excitation source equals to the amount of energy absorbed by the dissipation element. Despite the total power flow in the stiffened plate is slightly dependent on the integration path because of numerical error or singularity, it still can reveal the energy exchanges between the force and the damper as well as the amounts of energy transmitted through the stiffener and through the plate accurately.

The total power flow transmitted across any integration paths will be changed with the attached stiffeners of different cross-sectional geometry dimensions. It is observed that the amount of total energy flow is dependent on the natural vibration frequency of the whole stiffened plate structure. The higher the natural frequency of the coupled structure is, the smaller will the total energy be injected and transmitted, and the relationship between the total power flow through integration path 3 and the structural natural frequency is illustrated in Fig. 4. Although maximum change in the natural frequency is within 29% of the unstiffened plate, the amounts of total power flow through the cross section of structure are changed dramatically, as the dimensions of the stiffener get smaller.

As the amount of total injected and transmitted energy is model dependent, it is more appropriate to examine the ratio of the power flow through the pate to that through the stiffener. Figs. 5(a) and (b) display the variation of this ratio of power flow with the ratio of relative flexural rigidity of stiffener and plate according to integration path 3 and path 4, respectively. It is shown clearly that by increasing EI_s/D the relative percentage of power flowing through the cross section of the plate will be reduced while those through the stiffener will be increased. As EI_s/D increases, the relative power flow approaches a certain value. For EI_s larger than D, more than half of the total energy is transmitted through the stiffener from the source to the sink. If EI_s is smaller than D, most of the power flow will transmit through the plate. As the flexural rigidity of a stiffener varies as the third power of its height, an additional stiffener with $EI_s/D = 25$ is applied to further verify this changing regularity, the result is shown as the filled dots in Fig. 5(a).

3. Conclusions

The structural intensities of a rectangular plate with stiffeners attached have been calculated using the finite element method. The calculated results showed that stiffeners will localize the amount of energy flow in the plate while the affected regions are smaller compared with that without stiffeners attached, and the stiffener will change the orientations of energy flow in the plate in its vicinity. The total amount of injected or transmitted power flow is dependent on the natural vibration frequency of the whole structure, meanwhile, the relative percentage of power flowing through the cross section of the plate or the stiffener depends on the ratio of their flexural rigidities. By knowing these, an optimized design/modification or vibration control for a coupled stiffened plate system may be obtained based on the structural intensity concept.

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